1. PHD PROJECT DESCRIPTION (4000 characters max., including the aims and work plan)

Project title:

State parameter estimation and quantum entanglement tomography with randomised measurements

1.1. Project goals

Entanglement - quantum correlations having no analogue in the classical world has found many applications in quantum information protocols. Tools for detecting and certifying entanglement in a pair of particles produced by a source are of central importance for quantum-information applications.
Parties sharing an entaglement state can either perform measurments of many non-commuting quantum observables to estimate the density matrix of the state, and then apply various entanglement criteria to it or, if one can predict the entaglement state of the system, a single joint measurement can be applied to estimate the expected value of an entanglement witness.
Both these approaches assume control of the basis in which the projective measurement is performed. In more realistic scenarios, between the time instant when the joint state is born and the time instant when its parts reach the laboratories of the parties, it undergoes local dynamics, often not known. As the system is affected by a local unitary transformation, parties cannot trust their measurement basis settings, and they should average the obtained data with respect to the Haar measure of the local unitary group.
The aim of the project will be to identify the invariants of the local unitary group accessible in the randomised measurement scenario and the accessible parameters of the quantum state, in particular, those describing the non-classical correlations in the measured state.
Another goal will be to generalise the above to the non-unitary case of the local dynamics of subsystems, assuming a noise model is known.

1.2. Outline

The work involves collaboration with researchers from abroad. In particular, with Giovanni Scala from the Polytechnic University of Bari and Anindita Bera from the Birla Institute of Technology, Mesra. In the recent years they obtained in collaboration with the main suervisor some results concerning entanglement criteria, entanglement witnesses and their optimisation. Recently, they obtained some resuls concerning entanglement detection by randomised measurements.

1.3. Work plan

24 months:

- learn the grounds of quantum information theory, in partiular the quantum entaglement and methods of its detection and quantification.
- get familiar with performing nummerical experiment using scientific packages of Python language lige scipy and sage.
- ground of the theory of parameter estimation

24 months:

- theory of Lie groups and their invariant measures,
- basic representation theory, and the Weingarten calculus.
- theory of open system dynamics, noise models, group-covariant positive maps.

1.4. **Literature** *(max. 7 listed, as a suggestion for a PhD candidate preliminary study)*

- Michael A. Nielsen, Isaac L. Chuang Quantum Computation and Quantum Information. Cambridge University Press
- Entanglement witnesses: construction, analysis and classification J. Phys. A: Math. Theor. 47 483001 (2014)
- Entanglement Detection arXiv:0811.2803
- Quantum entanglement arXiv:quant-ph/0702225

- Analysing quantum systems with randomised measurements arXiv:2307.01251v2

1.5. Required initial knowledge and skills of the PhD candidate

Required knowledge: fundamentals of quantum mechanics and entanglement theory. Basic knowledge of programming, preferred language: Python
Welcomed: more advanced knowledge from quantum information theory, knowledge about group theory and representation theory.

1.6. Expected development of the PhD candidate's knowledge and skills

The project combines various aspects of quantum information science, group theory, representation theory, Haar measure on Lie groups, Wiengarted calculus, theory of parameter estimation, some elements of functional analysis, multilineal algebra and finally nummerical calculations. The student will gain and enhance knowledge of:

1. quantum entanglement detection,
2. entanglement distillation protocols, boud entanglement
3. quantum state estimation schemes,
4. theory of multiparticle entanglement,
5. theory of invariant integration on Lie groups, representation theory, Weigarten calculus, unitary designs
6. theory of open quantum systems, noise models

The candidate will obtain and extend the following skills:

1. Python programming and supporting research by veryfing hypotheses by nummerical experiments, fluency in using high-level python scientific packages and data visualisation tools,
2. formulating hypothesis and proving (disproving) them,
3. individual and group work, international cooperation,

4. presentation of scientific results in the form of articles, posters and conference presentations.