



## PHD PROJECT DESCRIPTION

(4000 characters max., including the aims and work plan to be published online)

**Project title:** Scalable Entanglement Detection with Randomised Measurements: Statistical inference and quantum circuit complexity for nonlinear invariant estimation

### 1.1. Project goals

Quantum entanglement is one of the most fundamental and non-classical features of quantum mechanics. It describes correlations between subsystems that cannot be explained by classical physics and constitutes a key resource for quantum technologies, including quantum computing, secure communication, and quantum simulation. A central practical challenge is how to detect and certify entanglement in real experiments, especially for systems involving many particles or high-dimensional states.

Standard methods, such as full quantum state tomography, require a number of measurements that grows fast with system size, making them infeasible beyond small systems. This motivates the search for scalable alternatives, which can extract relevant information about a quantum state without reconstructing it completely.

One promising approach is based on randomised measurements, where one performs sequences of simple, randomly chosen measurements on identical copies of a quantum system. By analyzing statistical correlations in the collected data, it is possible to estimate global properties of the state. In particular, recent research shows that one can access nonlinear functions of the quantum state, including quantities invariant under local unitary transformations. These nonlinear invariants capture intrinsic structural properties of quantum states and can be used to design powerful entanglement detection criteria that go beyond standard linear methods.

A crucial aspect of all protocols is that experimental data is always limited. Therefore, the project will incorporate finite-sample statistical inference, i.e., methods that allow one to draw reliable and quantifiable conclusions about entanglement from a finite number of measurements. This includes constructing estimators, deriving confidence bounds, and developing hypothesis tests adapted to nonlinear quantities.

From a practical perspective, randomised measurements are often implemented on programmable quantum platforms, such as IBM Quantum devices (e.g., *Eagle*, *Osprey*, *Heron*), as well as systems developed by Google, IonQ, Rigetti, or Quantinuum. In these settings, measurement protocols are realized using quantum circuits, i.e., sequences of quantum gates applied prior to measurement. The complexity of these circuits—including their depth, number of gates, and ability to approximate ideal random operations—plays a crucial role in determining whether a protocol is experimentally feasible. At the same time, similar ideas can also be implemented in non-circuit-based platforms (e.g., analog or photonic systems), which motivates a general analysis of resource requirements.

The main goal of this project is to develop a scalable and statistically rigorous framework for entanglement detection based on nonlinear invariant estimation using randomised measurements, combining tools from quantum information theory, modern statistics, and

quantum computing.

## 1.2. Outline

The project combines quantum information theory, statistical methods, and elements of quantum computing. It will:

- extend known entanglement criteria based on nonlinear invariants,
- design efficient randomised measurement protocols,
- develop statistically sound certification methods,
- analyse resource requirements of quantum implementations.

Applications include detection of entanglement in high-dimensional and many-body systems relevant for near-term quantum devices.

## 1.3. Work plan

### Year 1:

Introduction to quantum information and randomised measurements; reproduction of existing results; development of computational tools.

### Year 2:

Construction of new nonlinear invariant-based entanglement criteria; study of their detection power.

### Year 3:

Development of finite-sample statistical methods (confidence bounds, hypothesis testing); robustness analysis.

### Year 4:

Analysis of quantum circuit complexity and experimental feasibility; applications to realistic systems; thesis preparation.

## 1.4. Literature (max. 7 listed as a suggestion for a PhD candidate preliminary study)

1. R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki

*"Quantum entanglement"*

Rev. Mod. Phys. 81, 865 (2009)

arXiv: quant-ph/0702225

2. J. Elben, B. Vermersch, R. van Bijnen, C. Kokail, T. Brydges, C. Maier, M. Joshi, R. Blatt, C. F. Roos, P. Zoller

*"Renyi Entropies from Random Quenches in Atomic Hubbard and Spin Models"*

Phys. Rev. Lett. 120, 050406 (2018)

arXiv: 1709.05060

3. H.-Y. Huang, R. Kueng, J. Preskill



*"Predicting many properties of a quantum system from very few measurements"*

Nat. Phys. 16, 1050 (2020)

arXiv: 2002.08953

4. B. Vermersch, J. Elben, L. Knips, M. Brydges, C. Maier, R. Blatt, C. F. Roos, P. Zoller

*"Probing entanglement and quantum many-body states with randomized measurements"*

PRX Quantum 2, 020330 (2021)

arXiv: 2004.07213

5. I. Bengtsson, K. Życzkowski

*"Geometry of Quantum States: An Introduction to Quantum Entanglement"*

Cambridge University Press (2006, 2nd ed. 2017)

6. Giovanni Scala, Anindita Bera, Gniewomir Sarbicki

*"Entanglement detection via third-order local invariants from randomized measurements"*

Quantum Science and Technology 11, 015022 (2026)

arXiv: 2506.18303

7. Giovanni Scala, Gniewomir Sarbicki

*"Third-Order Local Randomized Measurements for Finite-size Entanglement Certification"*

arXiv: 2604.13165

### 1.5. Required initial knowledge and skills of the PhD candidate

- Master's degree in physics, mathematics, computer science, or related field
- Basics of quantum mechanics and linear algebra
- Some familiarity with probability/statistics
- Programming skills (e.g., Python, MATLAB)
- Interest in theoretical and computational research
- Ability to read scientific literature in English and work independently

### 1.6. Expected development of the PhD candidate's knowledge and skills

The candidate will gain expertise in:

- quantum information theory and entanglement
- modern statistical inference methods
- randomized algorithms and data analysis
- quantum computing and circuit models
- scientific programming and research communication

They will develop skills in independent research, problem-solving, and collaboration, preparing for careers in academia or quantum technologies.