

PHD PROJECT DESCRIPTION

(4000 characters max., including the aims and work plan to be published online)

Project title: Coxeter invariants in the representation theory of finite dimensional algebras

1.1. Project goals

Main goal of the project is to study the properties of the Coxeter transformation for finite dimensional algebras as well as the associated invariants: Coxeter polynomial, Coxeter spectrum, Coxeter numbers and related concepts. In particular, we plan to describe the general properties of these objects, to provide classifications of Coxeter invariants for distinguished classes of algebras and to study the properties of algebras and module categories encoded in these invariants.

1.2. Outline

Coxeter transformation associated with a finite dimensional algebra A of finite global dimension is a well-known invariant (up to similarity) of the derived equivalence, which encodes the behaviour of the Auslander-Reiten translation on the level of the Grothendieck group of A , see [ASS,H]. Its characteristic polynomial (called the *Coxeter polynomial*), its complex roots (forming the *Coxeter spectrum*), the spectral radius, and its order (called the *Coxeter number*, and related *reduced Coxeter numbers*) are useful derived invariants studied by many authors, see [L,LP,MP] and references therein. These studies show that the shapes of the Coxeter invariants reflect various specific properties of algebras and their module and derived categories. However, the exact nature of this relationship remains mysterious and not many general results are known (cf. [LP]). Even the explicit description of the Coxeter polynomial is known only for few classes of algebras, cf. [L,LP,JM].

Within this project we plan to study in a systematic manner these invariants, the properties of algebras and categories which they encode and possibly their interactions with other discrete invariants like the Euler homological form ([ASS,H]). We also plan to extend the study from the classical case of finite global dimension to Gorenstein algebras, much wider class for which the Coxeter transformation is well-defined. A systematic study of the Coxeter invariants in this more general context seems to be new in the literature and we hope to fill this gap. We plan to focus on the study of the Coxeter invariants for specific important classes of Gorenstein algebras like (skew-)gentle algebras [JM], self-injective algebras [SY], some Nakayama algebras [SY], cluster-tilted algebras and others.

If time permits, we also plan to go beyond module categories, namely, we hope to obtain some basic classification results for the respective generalization of the Coxeter formalism for certain triangulated categories with a Serre functor (cf. [H]).

1.3. Work plan

First year will be devoted to expanding general knowledge of the PhD candidate on the representation theory of finite dimensional algebras [ASS, H] and homological algebra. Moreover, we expect that the candidate will study in details particular classes of algebras like gentle algebras, Nakayama algebras, tilted algebras, and cluster-tilted algebras.

Our first research aims will be to obtain a description of the Coxeter invariants for specific non-trivial classes of algebras like gentle algebras, (Gorenstein) Nakayama algebras, self-injective algebras. Next we plan to embrace a bit „more complex” classes of Gorenstein algebras like skew-gentle algebras, cluster-tilted algebras or chosen classes of algebras of finite global dimension omitted in previous studies. We also want to produce a collection of examples and counterexamples for certain basic questions concerning the Coxeter invariants of Gorenstein algebras.

Using the experience gained in the study of concrete particular classes as above we plan to study e.g.:

- general properties of chosen Coxeter invariants,
- number-theoretic properties of the Coxeter polynomial like its palindromicity, cyclotomicity, irreducible factorizations, properties of roots etc.,
- some aspects in an abstract environment of bilinear lattices and (bi)graphs (cf. [L,MP]),
- properties of algebras and module categories encoded in the Coxeter invariants,
- Coxeter orbits in the Grothendieck group, their periodicity etc. (cf. [MP]),
- Euler homological form and its interplay with the Coxeter invariants.

In next stages, depending on time and progress, we hope to apply the techniques related with the Coxeter invariants in the study of derived and triangulated categories.

1.4. Literature (max. 7 listed as a suggestion for a PhD candidate preliminary study)

[ASS] I. Assem, D. Simson, and A. Skowroński, *Elements of Representation Theory of Associative Algebras 1: Techniques of Representation Theory*, London Math. Soc. Student Texts 65, Cambridge Univ. Press, Cambridge, New York, 2006.

[H] D. Happel, *Triangulated Categories in the Representation Theory of Finite Dimensional Algebras*, London Math. Soc. LNM 119, Cambridge Univ. Press, 1988.

[JM] J. A. Jiménez González and A. Mróz. A graph theoretic model for the derived categories of gentle algebras and their homological bilinear forms. preprint (37pp.), <https://arxiv.org/abs/2407.04817>, 2025.

[L] H. Lenzing. Coxeter transformations associated with finite dimensional algebras. *Computational Methods for Representations of Groups and Algebras*, Progress in Math. Birkhäuser, 173:287–308, 1999.

[LP] H. Lenzing and J. A. de la Peña. Spectral analysis of finite dimensional algebras and singularities. In *Trends in Representation Theory of Algebras and Related Topics*, pages 541–588. A. Skowroński, EMS Publishing House, Zürich, 2008

[MP] A. Mróz and J. A. de la Peña. Periodicity in bilinear lattices and the Coxeter formalism. *Linear Algebra Appl.*, 493:227–260, 2016.

[SY] A. Skowroński and K. Yamagata. *Frobenius Algebras I. Basic Representation Theory*. European Mathematical Society (EMS), Zürich, 2011.

1.5. Required initial knowledge and skills of the PhD candidate

We expect that the candidate knows basic notions and techniques of representation theory of finite dimensional algebras and quivers and category theory. Basic training in ring theory and homological algebra would be an advantage.

1.6. Expected development of the PhD candidate's knowledge and skills

After completing the project the PhD candidate will have a deep knowledge of representation theory of finite dimensional algebras. Moreover, the candidate will be able to find and solve research problems in mathematics.