PHD PROJECT DESCRIPTION

(4000 characters max., including the aims and work plan to be published online)

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Project title: Backward stochastic differential equations and their applications to optimization problems

1.1. Project goals

The project aims to develop existence and uniqueness results for backward stochastic differential equations (BSDEs), reflected BSDEs (RBSDEs), second-order BSDEs and second-order RBSDEs, and to apply them to optimization problems arising in mathematical finance, economics and engineering. It will study value functions and value processes, their regularity, stability and approximation, and optimal or epsilon-optimal strategies. Particular attention will be paid to Markovian models and links with Hamilton-Jacobi-Bellman partial differential equations.

1.2. Outline

BSDEs and RBSDEs give probabilistic representations for nonlinear expectations, optimal stopping, stochastic control, Dynkin games, impulse control and switching problems. The research will focus on selected topics: BSDEs and second-order BSDEs on general filtered spaces; RBSDEs and nonlinear optimal stopping; doubly reflected BSDEs and zero-sum Dynkin games; nonzero-sum Dynkin games, where the associated BSDE formulation remains open; second-order RBSDEs and robust control under model uncertainty; quasi-linear RBSDEs and impulse control; and multidimensional RBSDEs in random convex or non-convex sets with applications to switching problems.

1.3. Work plan

The first stage will consist of literature study and formulation of assumptions for selected BSDE/RBSDE classes. The second stage will develop existence, uniqueness, comparison, stability and approximation results. The third stage will apply these results to value processes/functions and analyze optimal or epsilon-optimal strategies. The final stage will address Markovian models, related HJB equations and preparation of publications.

1.4. Literature (max. 7 listed as a suggestion for a PhD candidate preliminary study)

1) Pardoux, E., Peng, S.: Adapted solution of a backward stochastic differential equation. Systems Control Lett. 14 (1990), 55-61.
2) Cvitanić, J., Karatzas, I.: Backward stochastic differential equations with reflection and Dynkin games. Ann. Probab. 24 (1996), 2024-2056.
3) El Karoui, N., Kapoudjian, C., Pardoux, E., Peng, S., Quenez, M.-C.: Reflected solutions of backward SDEs and related obstacle problems for PDEs. Ann. Probab. 25 (1997), 702-737.
4) Klimsiak, T.: Reflected BSDEs on filtered probability spaces. Stochastic Process. Appl. 125 (2015), 4204-4241.
5) Soner, H. M., Touzi, N., Zhang, J.: Wellposedness of second order backward SDEs. Probab. Theory Relat. Fields 153 (2012), 149-190.
6) Matoussi, A., Possamai, D., Zhou, C.: Second order reflected backward stochastic differential equations. Ann. Appl. Probab. 23 (2013), 2420-2457.
7) Klimsiak, T., Rzymowski, M.: Mokobodzki's intervals: an approach to Dynkin games when value process is not a semimartingale. arXiv:2407.15601 (2024).

1.5. Required initial knowledge and skills of the PhD candidate

Analytical thinking, readiness for self-study, understanding of mathematical analysis, solid knowledge of probability theory and functional analysis, and basic knowledge of stochastic processes and differential equations.

1.6. Expected development of the PhD candidate's knowledge and skills

The PhD candidate will develop the ability to conduct high-level research and present scientific results professionally, and will acquire advanced knowledge of stochastic equations, stochastic processes, optimization, stochastic analysis and selected links with PDEs.