1. PHD PROJECT DESCRIPTION (4000 characters max., including the aims and work plan)

Project title: Multi-state reliability systems with discrete component lifetimes

1.1. Project goals

The aim of this project is to study reliability properties of multistate systems consisting of multi-state components. The results will be obtained under the general assumption that all component lifetimes are discretely distributed random variables (rvs). Due to the discrete assumption the reliability analysis becomes more complicated. In the binary approach, more than one component may fail at the same time. If we extend the situation to the multi-state case, then with non-zero probability more than one component can enter the state 'j' at the same moment. In applications, we pay special attention to a geometric distribution among the discrete ones because of the memoryless property which rewers to the case when the past has no bearing on the future behavior. However, the theoretical results will be also useful for non-geometric case.

1.2. Outline

The focus of the project will be directed towards the concept of modeling technical devices (systems) with particular emphasis on the multi-state approach. So far, it has been considered in the literature from different perspectives. The reliability analysis of these systems has captured the attention of many researches in reliability engineering because of a potential impact on different fields. Several results are known to hold for binary systems, when both the system and its components can only work in one of two possible states: perfect functioning and the complete failure, and the weights of the units are the same. If we allow a wide range of performance levels of the system components, from perfect functioning to complete failure, then we are able to apply the multi-state approach. In real life situations it is more flexible because the system may not fail completely, but its efficiency can be much reduced.

1.3. Work plan

We are particularly interested in determining the properties of the rvs which represent the numbers of components of the multi-state system that are in particular states at the moment of the system failure. We focus our efforts on the situation when the component lifetimes are discretely distributed. Such assumption has become increasingly popular in the last five years in the literature. However, there have been only few papers restricted to the discrete case and devoted to multi-state modeling (see e.g. [1]). It follows from the fact that the discrete life time assumption makes the reliability analysis more complicated, because more than one

component may fail at the same time with non-zero probability. Moreover, it is more adequate especially when we are forced to monitor the system at some discrete points instead of the continuous inspections.

We consider these variates under various conditions. Among our research problems is to obtain conditional probabilities for rvs of our interest given some partial information about the status of the system. We start our investigations from the three-state **k**-out-of-*n* system, where the component lifetimes are independent and identically distributed (IID) rvs (cf. [1]). Next, we try to weaken these classical assumptions. On the one hand, our aim is to drop the independence assumption, on the other hand, the system can be composed of multiple types of components. We also plan to consider systems with more than three states or with other structures, e.g. the series-parallel systems, that is, the systems composed of disjoint parallel modules being serially connected. Finally, we are also interested in using the obtained results in the optimal design of multi-state system.

1.4. Literature *(max. 7 listed, as a suggestion for a PhD candidate preliminary study)*

[1] Goroncy A., Jasiński K. (2025), Discrete time three-state k-out-of-n system's failure and numbers of components in each state. *J. Comput. Appl. Math.* 457, 1–10, 116255.

[2] Huang J., Zuo M.J.,Wu Y. (2000). Generalized multi-state k-out-of-n:G systems. *IEEE Trans. Reliab.* 49:105–111.

[3] Eryilmaz S. (2014). Lifetime of multistate k-out-f-n systems. *Qual. Reliab. Eng. Int.* 30:1015–1022.

[4] Eryilmaz S. (2015). On the number of remaining components in three-state k-out-of-n system. *Oper. Res. Lett.* 43:616–621.

[5] Rausand M., Hoyland A. (2004). *System Reliability Theory. Models, Statistical Methods, and Applications*. Wiley, New Yersey.

[6] Tian Z, Zuo MJ, Yam RCM (2009). Multi-state k-out-of-n systems and their performance evaluation. *IIE Trans* 41:32–44.

1.5. Required initial knowledge and skills of the PhD candidate

PhD candidate should possess the following features:
- the ability and willingness to self-study,
- analytical thinking,

- a good knowledge of mathematics, with particular emphasis to the probability theory and mathematical statistics.
Moreover, a preliminary knowledge of the basis of the reliability theory would be an advantage.

1.6. Expected development of the PhD candidate's knowledge and skills

PhD candidate is expected to become a researcher in mathematics, who is able to study the literature, find the appropriate methods and apply them in the currently studied problem in order to obtain its solution. His presentation skills should also be developed, since during the studies he is expected to participate in scientific conferences and give talks. Most of all, his knowledge in statistical methods in reliability should increase and allow for further scientific development.